Comments on Dirichlet Branes at Angles

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Abstract

This paper illustrates the derivation of the low-energy background field solutions of D2-branes and D4-branes intersecting at non-trivial angles by solving *directly* the bosonic equations of motion of II supergravity coupled to a dilaton and antisymmetric fields. We also argue for how a similar analysis can be performed for any similar Dp-branes oriented at angles. Finally, the calculation presented here serves as a basis in the search for a systematic derivation of the background fields of the more general configuration of a p-brane 'angled' with a q-brane ($p \neq q$).

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1 Introduction

Since the realization by Polchinski that they are the carriers of Ramond-Ramond (RR) charges [1], D-branes contributed greatly to our understanding of the non-perturbative aspects of string theory. For example, recently bound states of D-branes have proven to be useful in the statistical interpretation of the Bekenstein-Hawking entropy associated with certain supersymmetric black holes in string theory [2]. In string theory, the large degeneracy of a bound state can be counted reliably at weak coupling. Then as one increases the strength of the coupling, the bound state eventually undergoes a gravitational collapse and forms a black hole of string theory. In these analyses the bound states of interest must be supresymmetric, *i.e.,* saturate a BPS bound, since the spectrum of supersymmetric states are protected against loop corrections as we vary the coupling. Therefore the degeneracy of the black holes states at strong coupling must be the same as that of the corresponding weakly coupled D-brane bound state[1]. This is one of the reasons that has made supersymmetric D-brane bound states a subject of growing interest.

Recently a number of BPS-saturated configurations representing bound states of various Dp-brane solutions of type II string theory were obtained [4]. In all these bound state solutions the constituent Dp-branes are either parallel to each other or intersect orthogonally according to the harmonic superposition rules where each brane is specified by an independent harmonic function [5]. That these cannot be the only possibilities an that there exist supresymmetric bound states where the D-branes intersect at angles which are different from zero and $\pi/2$ was first pointed out in [6]. There it was shown that configurations of multiple D-branes related by $SU(N)$ rotations will preserve unbroken supersymmetry. The work of [6] was later extended in [7] where it was demonstrated that there are toroidal compactifications of D-branes at angles in type II string theory that are supersymmetric only when non-trivial antisymmetric tensor moduli fields are turned on at the position of the D-brane. In both [6] and [7], the intervening configurations of D-branes at angles were analyzed using a string calculation. It is natural to try to demonstrate the findings of [6] and [7] from the point of view of the classical D-brane solution of supergravity.

In effect, some research was conducted lately in which the classical solutions of branes oriented at angles appeared. We do not review any of these solutions here and refer the reader to the work in [8,9]. For our present purpose, however, it suffices to notice that in the discussion of both [8] and [9], the authors started with a well known brane configuration and then used an appropriate combination of boosts, $T-$duality and $S-$duality transfomations to generate the supresymmetric bound state of D-branes at angles. In this paper, we will take a different approach and show how the low-energy background fields of D2-branes and D4-branes whose pairs are oriented at angles can be derived by solving *directly* the bosonic equations of motion of II supergravity coupled to a dilaton and antisymmetric fields.

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[1]Recently in [3], however, a similar counting of states was performed for a certain class of *non-supresymmetric* but extremal black holes. In this case, supersymmetry alone does not protect the spectrum and yet the degeneracy at strong coupling was shown to be the same as that at weak coupling.

The paper is organized as follows: We start by establishing our conventions in Section (2), as well as presenting the low-energy action for type IIA string theory and the equations of motion derived from it. In Section (3), by restricting to the case where only the R-R 4-form field strength is kept excited, we study in detail the configuration of two D2-branes by $SU(2)$ angles, and which preserves $1/4$ of supersymmetry. In particular, we discuss the choice of the ansätze to be made for the background fields in order to solve the equations of motion. In Section (4), we carry out the actual solving of the equations of motion and present all the calculational techniques that allow us to find the field solutions. We consider in Section (5) the case of D4-branes where by also solving directly the equations of motion we find the low-energy solution of the corresponding bound state with angle and with $1/4$ unbroken supresymmetry. We also argue how a similar analysis can be performed for the general case of p-branes oriented at angles in type II string theories and M-theory. Finally, a brief discussion follows in Section (6).

2 Some Preliminaries

The bosonic part of the low-energy action for type IIA string theory in ten dimensions is [10]

$$
\begin{aligned}
S_{IIA} \;=\; & \frac{1}{16\pi G_N} \int d^{10}x \sqrt{-G} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\left(\frac{1}{3!}\,e^{-\phi}\,H^2 - \frac{1}{2!}\,e^{3\phi/2}\,f^2 - \frac{1}{4!}\,e^{\phi/2}\,F^2 \right) \right] \\
& - \frac{1}{32\,\pi\,G_N} \int B\,dA\,dA\,,
\end{aligned} \tag{1}
$$

where $G_{\mu\nu}$ is the Einstein-frame metric, $H = dB$ is the 3-form field strength of the Kalb-Ramond NS-NS field, $f = da$ and $F = dA - Ha$ are the 2-form and 4-form R-R field strengths, and finally ϕ is the dilaton which is taken to vanish asymptotically.

Dropping the various Chern-Simons terms from S_{IIA} (since the solutions we will present throughout all the paper are also consistent solutions of the full action), and by setting both the field strength H and f to zero the field equations of motion take the simple form[2]

$$
R^{\mu}{}_{\nu} = \frac{1}{2}\,\partial^{\mu}\phi\,\partial_{\nu}\phi + \frac{1}{2\,4!}\,e^{\phi/2}\left[4\,F^{\mu\alpha_2\alpha_3\alpha_4}\,F_{\nu\alpha_2\alpha_3\alpha_4} - \frac{3}{8}\,G^{\mu}{}_{\nu}\,F^2 \right]\,, \tag{2}
$$

$$
\Box\phi = \frac{1}{4\,4!}\,e^{\phi/2}\,F^2\,, \tag{3}
$$

$$
\partial_{\mu_1}\left(\sqrt{-G}\,e^{\phi/2}\,F^{\mu_1\mu_2\mu_3\mu_4} \right) = 0\,. \tag{4}
$$

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[2]Our conventions are $(-,+,\cdots,+)$ signature for the metric $G_{\mu\nu}$, for the Riemann tensor we take $R^{\lambda}{}_{\mu\nu_{\kappa}} = \partial_{\nu}\,\Gamma^{\lambda}_{\mu\,\kappa} - \partial_{\kappa}\,\Gamma^{\lambda}_{\mu\,\nu} + \Gamma^{\eta}_{\mu\,\kappa}\,\Gamma^{\lambda}_{\nu\,\eta} - \Gamma^{\eta}_{\mu\,\nu}\,\Gamma^{\lambda}_{\kappa\,\eta}$ and for the affine connection we take $\Gamma^{\lambda}_{\mu\,\nu} = G^{\lambda\rho}\left(\partial_{\mu}\,G_{\rho\,\nu} + \partial_{\nu}\,G_{\mu\,\rho} - \partial_{\rho}\,G_{\mu\,\nu} \right)/2$.

The statement that the R-R 4-form F is the field strength of a 3-form potential A is equivalent to the Bianchi identity imposed by the condition:

$$\partial_{[\nu} F_{\mu_1 \mu_2 \mu_3 \mu_4]} = 0 \,. \tag{5}$$

In general, in type II superstring theories there are two kind of p-branes, those charged under NS-NS fields and those carrying R-R charge. The first ones contain the usual elementary (perturbative) string states in the case of 1-brane [11] and the purely solitonic objects in the case of the 5-brane [12]. For the second ones however, the branes have been shown to describe Dp-branes [1]. Within type IIA theory, the Dp-brane can have $p = 0, 2, 4, 6, 8$, while for type IIB theory p ranges over $-1, 1, 3, 5, 7, 9$. Since only the R-R 4-form field strength F is kept excited in our type IIA theory, then we are certain that the low-energy background fields that solve the equations of motion derived from S_{IIA} are those of D2-branes, *i.e.,* D-membranes. The D2-branes are *electrically* charged.

Our purpose in this paper is to find the low-energy background fields describing two D2-branes intersecting at angles different from zero and $\pi/2$ by solving *directly* the equations of motion (2,3,4,5). Recently using a similar strategy, a model-independent derivation for the general rules of orthogonnally intersecting extreme branes in arbitrary spacetime dimension D was given in [13]. Following the same approach, the non-extreme case was also treated in [14]. By specializing to the branes occuring in type II superstring theories and in M-theory, the intersection rules found in this way are compatible with the harmonic superposition rules for intersecting extreme p-branes formulated originallly by Tseytlin in [5] and later generalized to non-extreme cases in [14] (see also [15]).

3 The Making of Two D2-Branes at Angles

To obtain a brane configuration that correspond to two D2-branes intersecting at some non-trivial angle $\theta \neq 0, \pi/2$, we can start by two parallel D2-branes oriented both along the $x^{1,2}-$axes. Then we rotate one of them by the angle θ in the $x^{1,3}$ plane and $-\theta'$ in the $x^{2,4}$ palne using a rotation matrix which acts on the coordinates $x^{1,2,3,4}$ in the following way:

$$
\begin{aligned}
y^1 &= \cos\theta \, x^1 - \sin\theta \, x^3 \,, & y^3 &= \sin\theta \, x^1 + \cos\theta \, x^3 \,, \\
y^2 &= \cos\theta \, x^2 + \sin\theta \, x^4 \,, & y^4 &= -\sin\theta \, x^2 + \cos\theta \, x^4 \,.
\end{aligned}
\tag{6}
$$

In the new coordinates $y^{1,2,3,4}$ defined above, one brane stays oriented along the $x^{1,2}-$axes while the other is now oriented along the $y^{1,2}-$axes. From the work of [6], the new configuration obtained will preserve some supresymmetry (1/4 in the present case) if the relative orientation of the D2-branes are restricted by an $SU(2)$ rotation. If one defines the complex coordinates $z^1 = x^1 + ix^3$ and $z^2 = x^2 + ix^4$, then the rotations in (6) take the simple form $(z^1 \,, z^2) \to \left(e^{i\theta} z^1 \,, e^{-i\theta'} z^2 \right)$. It follows that if the D2-branes should be related by

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an $SU(2)$ rotation, *i.e.*, $z^i \rightarrow [\exp{(i\theta\,\sigma_3)}]^i{}_j\, z^j$, then $\theta = \theta'$ [9]. From this construction it is clear that the general supergravity solution of D2-branes at angles is in effect an interpolation between the parallel case ($\theta = 0$) and the orthognal intersection case ($\theta = \pi/2$). This observation will prove to be useful in section (4) when we come to inetgrate the field equations of (2,3,4,5). The reason for this is that eventhough the equations of motion of are the simplest possible since they involve only the fields ϕ, $G_{\mu\nu}$ and $F_{\mu_1\mu_2\mu_3\mu_4}$, it would not be possible to solve them directly without having recourse to make few ansätze. To build our intuition on the kind of ansätze that one should be making, we review below the supergravity solutions of two parallel D2-branes, two D2-branes intersecting orthogonally over a point and a single rotated D2-brane.

3.1   Parallel D2-Branes

In the Einstein-frame metric, the low-energy background field solution [16,17] describing two parallel D2-branes oriented along $x^{1,2}-$axes contains only a non-trivial metric, dilaton and a single R-R potential, A as follows:

$$
\begin{aligned}
\mathrm{d}s^2 &= (1 + h_1 + h_2)^{\frac{3}{8}} \left(\frac{-\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2}{1 + h_1 + h_2} + \left(\mathrm{d}x^3\right)^2 + \left(\mathrm{d}x^4\right)^2 + \cdots + \left(\mathrm{d}x^9\right)^2 \right), \\
A &= \frac{h_1 + h_2}{1 + h_1 + h_2} \mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2\,, \\
F_{t12i} &= -\left(\frac{1}{1 + h_1 + h_2}\right)^2 \partial_i h_1 - \left(\frac{1}{1 + h_1 + h_2}\right)^2 \partial_i h_2\,, \\
e^{2\phi} &= \sqrt{1 + h_1 + h_2}\,.
\end{aligned}
\tag{7}
$$

The spatial coordinates (x^1, x^2) are parallel to the worldvolume of the D2-branes, while the orthogonal space is spanned by the coordinates x^i with $i = 3, 4, ..., 9$. A noteworthy feature of the above solution is that it is completely specified by the harmonic functions $h_{1,2}$ which solve the flat space Poisson's equation in the transverse space:

$$
\delta^{ij}\, \partial_i\, \partial_j\, h_{1,2} = 0\,.
\tag{8}
$$

The solutions of (8) has a dependence only through the radius $r^2 = \sum_{i=3}^{9} (x^i)^2$ in the transverse space and is given by:

$$
h_{1,2} \equiv h_{1,2}(r) = \frac{c_{1,2}\, Q_{1,2}}{\left|\vec{r} - \vec{r}_{1,2}\right|^5}\,,
\tag{9}
$$

where $\vec{r}_1$ is the location in the transverse space of the first D2-brane and $\vec{r}_2$ is the location of the second.

3.2 Orthognal D2-Branes

In the Einstein-frame metric, the low-energy background field solution [5,13] describing two orthogonal D2-branes intersecting over a point, where one of them is oriented along $x^{1,2}-$axes and the other one oriented along $x^{3,4}-$axes is given by:

$$
\begin{aligned}
ds^2 &= (1 + h_1 + h_2 + h_1\,h_2)^{\frac{3}{8}} \left(\frac{-dt^2}{1 + h_1 + h_2 + h_1\,h_2} + \frac{(dx^1)^2 + (dx^2)^2}{1 + h_1} \right.\\
&\qquad \left. + \frac{(dx^3)^2 + (dx^4)^2}{1 + h_2} + \left(dx^5\right)^2 + \cdots + \left(dx^9\right)^2 \right),\\
A &= \frac{h_1}{1 + h_1}\,dt \wedge dx^1 \wedge dx^2 - \frac{h_2}{1 + h_2}\,dt \wedge dx^3 \wedge dx^4\\
F_{t12i} &= -\left(\frac{1}{1+h_1}\right)^2 \partial_i h_1\,,\\
F_{t34i} &= \left(\frac{1}{1+h_2}\right)^2 \partial_i h_2\,,\\
e^{2\phi} &= \sqrt{1 + h_1 + h_2 + h_1\,h_2}\,.
\end{aligned}
\tag{10}
$$

For this solution, the harmonic functions $h_{1,2}$ satisy the Poisson's equation in the transverse subspace spanned by $(x^5, x^6, x^7, x^8, x^9)$:

$$
\delta^{ij}\,\partial_i\,\partial_j\,h_{1,2} = 0 \qquad \text{for} \quad i = 5, \cdots, 9\,,
\tag{11}
$$

yielding the solutions

$$
h_{1,2} \equiv h_{1,2}(r) = \frac{c_{1,2}\,Q_{1,2}}{|\vec{r} - \vec{r}_{1,2}|^3}\,,
\tag{12}
$$

where here $r^2 = \sum_{i=5}^{9} (x^i)^2$.

3.3 A Single Rotated D2-Brane

In (7) or (10), if we set $h_2 = 0$ we recover the background field solution of a single D2-Brane oriented along the $x^{1,2}-$axes. Now if we perform a rotation to orient this D2-brane along the $y^{1,2}-$axes which were introduced in (6), then the new configuration is given by:

$$
\begin{aligned}
ds^2 &= (1 + h_1)^{\frac{3}{8}} \left(\frac{-dt^2}{1 + h_1} + \frac{1 + h_1 \sin^2\theta}{1 + h_1} \left[\left(dx^1\right)^2 + \left(dx^2\right)^2\right] \right.\\
&\qquad + \frac{1 + h_1 \cos^2\theta}{1 + h_1} \left[\left(dx^3\right)^2 + \left(dx^4\right)^2\right] + 2\,\cos\theta\,\sin\theta\,\frac{h_1}{1 + h_1} \left(dx^1\,dx^3 - dx^2\,dx^4\right)\\
&\qquad \left. + \left(dx^5\right)^2 + \cdots + \left(dx^9\right)^2 \right),
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{h_1 \cos^2 \theta}{1 + h_1} \, dt \wedge dx^1 \wedge dx^2 + \frac{h_1 \cos \theta \sin \theta}{1 + h_1} \left(dt \wedge dx^2 \wedge dx^3 + dt \wedge dx^1 \wedge dx^4 \right) \\
&\quad - \frac{h_1 \sin^2 \theta}{1 + h_1} \, dt \wedge dx^3 \wedge dx^4 \,, \\
e^{2\phi} &= \sqrt{1 + h_1} \,.
\end{aligned} \tag{13}
$$

The D2-brane above is originally oriented along the $x^{1,2}-$axes, and normally, we would choose the harmonic functions $h_{1,2}$ to be on the whole tranverse space $(x^3, x^4, \cdots, x^9)$ as in (9). For the present solution (13), however, since the D2-brane is delocalized in the $x^{3,4}$ coordinates the harmonic functions $h_{1,2}$ are given rather by (12).

Next we display the field strengh which couples to this 'tilted' D2-brane. A straightforward calculation using the R-R 3-form potential A given in (13) yields:

$$
\begin{aligned}
F_{t12i} &= -\left(\frac{\cos \theta}{1 + h_1} \right)^2 \partial_i h_1 \equiv -\left(\frac{\mathcal{A}}{1 + h_1} \right)^2 \partial_i h_1 \,, \\
F_{t23i} &= F_{t14i} = -\left(\frac{\cos \theta}{1 + h_1} \right) \left(\frac{\sin \theta}{1 + h_1} \right) \partial_i h_1 \equiv -\left(\frac{\mathcal{C}}{1 + h_1} \right)^2 \partial_i h_1 \,, \\
F_{t34i} &= \left(\frac{\sin \theta}{1 + h_1} \right)^2 \partial_i h_1 \equiv \left(\frac{\mathcal{B}}{1 + h_1} \right)^2 \partial_i h_1 \,,
\end{aligned} \tag{14}
$$

where $\partial_i h_1 = \partial h_1 / \partial x^i$, for $i = 5, \cdots, 9$. A glance at the expressions of the field strength above and the expression of the metric in (13) reveals the following relations:

$$
\begin{aligned}
\mathcal{C}^2 &= \mathcal{A} \mathcal{B} \,, \tag{15} \\
G_{11} &= G_{22} = \frac{1 + h_1 \sin^2 \theta}{1 + h_1} \,, \tag{16} \\
G_{33} &= G_{44} = \frac{1 + h_1 \cos^2 \theta}{1 + h_1} \,, \tag{17} \\
G_{13} &= -G_{24} = A_{t23} = A_{t14} = \cos \theta \sin \theta \, \frac{h_1}{1 + h_1} \,. \tag{18}
\end{aligned}
$$

These relations will prove their worth below when we come to discuss the choice of the necessary ansätze involved in the resolution of the equations of motion to find the field solution of the two D2-branes at angles.

3.4   The Choice of the Ansätze

To choose an ansätz for the background fields $(G_{\mu\nu}, F_{\mu_1 \mu_2 \mu_3 \mu_4}, \phi)$ of the bound state of two D2-branes at angles, we refer back to the construction of this configuration proposed at the beginning of Section (3). Following this construction, we take one of the D2-branes, which we represent by the harmonic function h_1, to be oriented along the $y^{1,2}-$axes and the

other one, which we represent by the harmonic function h_2, is set parallel to the $x^{1,2}$−axes. (The $y^{1,2}$ coordinates are the same as defined in (6).) This way of representing the D2-branes at angles has the advantage of yielding the well known supregravity solution of Section (3.3) when $h_2 = 0$. Based on these simple remarks and taking advantage of the relations of (16,17,18), we suggest the following starting ansätz for the background fields of two 'angled' D2-branes expressed in the Einstein-frame metric:

$$
\begin{aligned}
\mathrm{d}s^2 &= -U^2\mathrm{d}t^2 + G_{11}\left(\left(\mathrm{d}x^1\right)^2 + \left(\mathrm{d}x^2\right)^2\right) + G_{33}\left(\left(\mathrm{d}x^3\right)^2 + \left(\mathrm{d}x^4\right)^2\right) \\
&\quad + 2\,G_{13}\left(\mathrm{d}x^1\,\mathrm{d}x^3 - \mathrm{d}x^2\,\mathrm{d}x^4\right) + V^2\left(\mathrm{d}x^i\,\mathrm{d}x_i\right), \\
F_{t12i} &= -\left(\frac{\mathcal{A}_1}{E}\right)^2 \partial_i h_1 - \left(\frac{\mathcal{A}_2}{E}\right)^2 \partial_i h_2, \\
F_{t23i} &= F_{t14i} = -\left(\frac{\mathcal{C}_1}{E}\right)^2 \partial_i h_1 + \left(\frac{\mathcal{C}_2}{E}\right)^2 \partial_i h_2, \\
F_{t34i} &= \left(\frac{\mathcal{B}_1}{E}\right)^2 \partial_i h_1 + \left(\frac{\mathcal{B}_2}{E}\right)^2 \partial_i h_2, \\
e^{2\phi} &= \sqrt{E}, \tag{19}
\end{aligned}
$$

where

$$
\begin{aligned}
U &\equiv U\left(h_1, h_2, \theta\right), \quad V \equiv V\left(h_1, h_2, \theta\right), \quad E \equiv E\left(h_1, h_2, \theta\right), \\
G_{11} &\equiv G_{11}\left(h_1, h_2, \theta\right), \quad G_{13} \equiv G_{13}\left(h_1, h_2, \theta\right), \quad G_{33} \equiv G_{33}\left(h_1, h_2, \theta\right), \\
\mathcal{A}_m &\equiv \mathcal{A}_m\left(h_1, h_2, \theta\right), \quad \mathcal{B}_m \equiv \mathcal{B}_m\left(h_1, h_2, \theta\right), \quad \mathcal{C}_m \equiv \mathcal{C}_m\left(h_1, h_2, \theta\right), \quad m = 1, 2, \tag{20}
\end{aligned}
$$

and the harmonic functions are $h_{1,2} = c_{1,2}\,Q_{1,2}/|\vec{r} - \vec{r}_{1,2}|^3$, with $r^2 = \sum_{i=5}^{9}(x^i)^2$.

All the unknown functions entering in the definition of the fields above are represented by the set $(U, V, G_{11}, G_{13}, G_{33}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2, \mathcal{C}_1, \mathcal{C}_2, \phi)$ and our task in what follows will be to solve for them using the equations of motion. As we shall now discuss, the number of unknowns in this set can be reduced further by making further ansätze. First of all, let us note that (by construction) if we set $\theta = 0$ our bound state must reduce to that of Section (3.1) where both the D2-branes are lying in the (x^1, x^2) plane and delocalized in the $x^{3,4}$−directions. Similarly, the configuration of two orthogonally D2-branes of Section (3.2) is reproduced by choosing $\theta = \pi/2$. Finally, the special case $h_2 = 0$ corresponds to the 'tilted' D2-brane of Section (3.3). Because of these correspondences, we define our first simplifying ansätze, which accompany the low-energy solution above, by imposing the following constraints:

$$
(\mathcal{C}_1)^2 = \mathcal{A}_1\,\mathcal{B}_1, \qquad (\mathcal{C}_2)^2 = \mathcal{A}_2\,\mathcal{B}_2, \tag{21}
$$

$$
\partial_1^2\,E = \partial_2^2\,E = 0, \tag{22}
$$

where $\partial_{1,2}E \equiv \partial E/\partial h_{1,2}$. Thus, we can now drop from our list of unknowns the functions $\mathcal{C}_1$ and $\mathcal{C}_2$.

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The third ansätz that we will make to reduce even further the number of unknowns is related to the condition of extremality. The requirement that the configuration of the D2-branes at angles preserves 1/4 of supresymmetries translates into the following condition on the metric components:

$$U \left(G_{11} G_{33} - G_{13}^2\right) V^3 = 1 \,. \tag{23}$$

This equation is a generalization of the extremality condition used in [13] for the case of a bound state of two orthogonal D-branes. With this ansätz our list of unknown functions has reducedto $(U, G_{11}, G_{13}, G_{33}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2, \phi)$. We are now ready to start solving the equations of motion using the ansätze introduced in this section.

4   Solving the Equations of Motion

Using the ansätz solution of (19) accompanied by the condition in (21), the equations of motion of (2,3,4) simplify considerably and become:

$$\frac{1}{4!} e^{\phi/2} F^2 = - V^{-2} \left(S_{11} \left(\partial_i h_1\right)^2 + S_{22} \left(\partial_i h_2\right)^2 + 2 S_{12} \left(\partial_i h_1\right) \left(\partial_i h_2\right)\right) , \tag{24}$$

$$\partial_i \partial_i \ln U = \frac{5}{16} \left[S_{11} \left(\partial_i h_1\right)^2 + S_{22} \left(\partial_i h_2\right)^2 + 2 S_{12} \left(\partial_i h_1\right) \left(\partial_i h_2\right)\right], \tag{25}$$

$$\partial_i \partial_i \ln \left(G_{11} G_{33} - G_{13}^2\right) = \frac{1}{4} \left[S_{11} \left(\partial_i h_1\right)^2 + S_{22} \left(\partial_i h_2\right)^2 + 2 S_{12} \left(\partial_i h_1\right) \left(\partial_i h_2\right)\right], \tag{26}$$

$$\partial_i \partial_i \phi = -\frac{1}{4} \left[S_{11} \left(\partial_i h_1\right)^2 + S_{22} \left(\partial_i h_2\right)^2 + 2 S_{12} \left(\partial_i h_1\right) \left(\partial_i h_2\right)\right], \tag{27}$$

$$\partial_i \ln U \, \partial_j \ln U - \frac{1}{2} \left(\partial_i G^{11} \, \partial_j G_{11} + \partial_i G^{33} \, \partial_j G_{33} + 2 \partial_i G^{13} \, \partial_j G_{13}\right) + 3 \partial_i \ln V \, \partial_j \ln V$$

$$+ \delta_{ij} \, \partial_k \partial_k \ln V = -\frac{1}{2} \partial_i \phi \, \partial_j \phi - \frac{3}{16} \delta_{ij} \left[S_{11} \left(\partial_k h_1\right)^2 + S_{22} \left(\partial_k h_2\right)^2 + 2 S_{12} \partial_k h_1 \partial_k h_2\right]$$

$$+ \frac{1}{2} \left[S_{11} \partial_i h_1 \partial_j h_1 + S_{22} \partial_i h_2 \partial_j h_2 + S_{12} \partial_i h_1 \partial_j h_2 + S_{12} \partial_i h_2 \partial_j h_1\right] , \tag{28}$$

with

$$S_{12} = \frac{\left(E^{31/16} \, U\right)^{-2}}{\left(G_{11} \, G_{33} - G_{13}^2\right)^2} \left[G_{33} \, \mathcal{A}_1 \, \mathcal{A}_2 - G_{11} \, \mathcal{B}_1 \, \mathcal{B}_2 - G_{13} \left(\mathcal{A}_1 \, \mathcal{B}_2 - \mathcal{A}_2 \, \mathcal{B}_1\right)\right]^2 , \tag{29}$$

$$S_{11} = \frac{\left(E^{31/16} \, U\right)^{-2}}{\left(G_{11} \, G_{33} - G_{13}^2\right)^2} \left[G_{33} \, \mathcal{A}_1^2 + G_{11} \, \mathcal{B}_1^2 + 2 \, G_{13} \, \mathcal{A}_1 \, \mathcal{B}_1\right]^2 , \tag{30}$$

$$S_{22} = \frac{\left(E^{31/16} \, U\right)^{-2}}{\left(G_{11} \, G_{33} - G_{13}^2\right)^2} \left[G_{33} \, \mathcal{A}_2^2 + G_{11} \, \mathcal{B}_2^2 - 2 \, G_{13} \, \mathcal{A}_2 \, \mathcal{B}_2\right]^2 . \tag{31}$$

8

As an immediate consequence of the equations (25,26,27) and the extremality condition (23), we have

$$U = E^{-5/16}, \quad V = E^{3/16}, \quad \left(G_{11}\,G_{33} - G_{13}^2\right) = E^{-1/4}. \tag{32}$$

These relations simplify furher our problem and allows us to keep only the set of functions $(G_{11}, G_{13}, G_{33}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2)$. in our list of unknowns.

Since the dilaton ϕ has been dropped from our list of unknowns we will in a first step determine the expressions of the metric components G_{11}, G_{13} and G_{33} in terms of it. For this, we begin by showing using the harmonic property of the functions $h_{1,2}$, *i.e.*, $\partial_i\,\partial_i\,h_{1,2} = 0$, and the relation $4\,\phi = \ln E$ from the ansätz in (19) that:

$$4\,\partial_i\partial_i\phi = -\left(\frac{\partial_1 E}{E}\right)^2 (\partial_i h_1)^2 - \left(\frac{\partial_2 E}{E}\right)^2 (\partial_i h_2)^2 - 2\frac{\partial_1 E\partial_2 E - E\partial_1\partial_2 E}{E^2}\,(\partial_i h_1)\,(\partial_i h_2), \tag{33}$$

where we used the ansätz (22) to set $\partial_1^2 E = \partial_2^2 E = 0$. Comparing equation (33) with the equation of motion of ϕ in (27) and using the relations (29,30,31) for S_{11}, S_{13} and S_{33} after substituting also by the expressions in (32), we obtain:

$$E^{3/4}\left(\partial_1 E\partial_2 E - E\partial_1\partial_2 E\right) = \left[G_{33}\mathcal{A}_1\mathcal{A}_2 - G_{11}\mathcal{B}_1\mathcal{B}_2 - G_{13}\left(\mathcal{A}_1\mathcal{B}_2 - \mathcal{A}_2\mathcal{B}_1\right)\right]^2, \tag{34}$$

$$E^{3/4}\left(\partial_1 E\right)^2 = \left[G_{33}\,\mathcal{A}_1^2 + G_{11}\,\mathcal{B}_1^2 + 2\,G_{13}\,\mathcal{A}_1\,\mathcal{B}_1\right]^2, \tag{35}$$

$$E^{3/4}\left(\partial_2 E\right)^2 = \left[G_{33}\,\mathcal{A}_2^2 + G_{11}\,\mathcal{B}_2^2 - 2\,G_{13}\,\mathcal{A}_2\,\mathcal{B}_2\right]^2. \tag{36}$$

Now if we take the square root of the three equations above we are led remarkably to a simple system of three simultaneous linear equations where we can at last solve for G_{11}, G_{13} and G_{33} in terms of $(\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2, E)$. Thus, we have

$$E^{3/8}\sqrt{\partial_1 E\partial_2 E - E\partial_1\partial_2 E} = G_{33}\mathcal{A}_1\mathcal{A}_2 - G_{11}\mathcal{B}_1\mathcal{B}_2 - G_{13}\left(\mathcal{A}_1\mathcal{B}_2 - \mathcal{A}_2\mathcal{B}_1\right), \tag{37}$$

$$E^{3/8}\,\partial_1 E = G_{33}\,\mathcal{A}_1^2 + G_{11}\,\mathcal{B}_1^2 + 2\,G_{13}\,\mathcal{A}_1\,\mathcal{B}_1, \tag{38}$$

$$E^{3/8}\,\partial_2 E = G_{33}\,\mathcal{A}_2^2 + G_{11}\,\mathcal{B}_2^2 - 2\,G_{13}\,\mathcal{A}_2\,\mathcal{B}_2. \tag{39}$$

Solving for the metric components we find:

$$G_{13} = \frac{E^{3/8}\left[(\partial_1 E)\,\mathcal{A}_2\mathcal{B}_2 - (\partial_2 E)\,\mathcal{A}_1\mathcal{B}_1 - \sqrt{\partial_1 E\partial_2 E - E\partial_1\partial_2 E}\,\left(\mathcal{A}_1\mathcal{B}_2 - \mathcal{A}_2\mathcal{B}_1\right)\right]}{\left(\mathcal{A}_1\mathcal{B}_2 + \mathcal{A}_2\mathcal{B}_1\right)^2},$$

$$G_{11} = \frac{E^{3/8}\left[(\partial_1 E)\,(\mathcal{A}_2)^2 + (\partial_2 E)\,(\mathcal{A}_1)^2 - 2\,\sqrt{\partial_1 E\partial_2 E - E\partial_1\partial_2 E}\,\mathcal{A}_1\mathcal{A}_2\right]}{\left(\mathcal{A}_1\mathcal{B}_2 + \mathcal{A}_2\mathcal{B}_1\right)^2},$$

$$G_{33} = \frac{E^{3/8}\left[(\partial_1 E)\,(\mathcal{B}_2)^2 + (\partial_2 E)\,(\mathcal{B}_1)^2 + 2\,\sqrt{\partial_1 E\partial_2 E - E\partial_1\partial_2 E}\,\mathcal{B}_1\mathcal{B}_2\right]}{\left(\mathcal{A}_1\mathcal{B}_2 + \mathcal{A}_2\mathcal{B}_1\right)^2}. \tag{40}$$

It is worth pointing out that such simplification was possible only because the coefficients S_{11}, S_{12} and S_{22} appear all in the form of a complete square which is a consequence of

imposing the ansätz (21). In other words, even if we have not imposed the ansätz (21) at the beginning we would have been driven to it by insisting that S_{11}, S_{13} and S_{33} are each in a form of a complete square in order to obtain at the end a solvable linear system of equations.

The last simplification occuring in our problem uses the Bianchi identity on the R-R field strength of the configuration and which effectively relates the functions $\mathcal{A}_1$ and $\mathcal{A}_2$ to each other and similarly for the functions $\mathcal{B}_1$ and $\mathcal{B}_2$. That is we have:

$$\left(\frac{\mathcal{A}_1}{E}\right)^2 = \partial_1 A_{t12}\,, \qquad \left(\frac{\mathcal{A}_2}{E}\right)^2 = \partial_2 A_{t12}\,, \tag{41}$$

$$\left(\frac{\mathcal{B}_1}{E}\right)^2 = -\partial_1 A_{t34}\,, \qquad \left(\frac{\mathcal{B}_2}{E}\right)^2 = -\partial_2 A_{t34}\,, \tag{42}$$

where $A_{\mu_1\mu_2\mu_3}$ is the 3-form potential an $\partial_{1,2} \equiv \partial/\partial h_{1,2}$. Therefore, the low-energy solution of D2-branes at angles is completely determined by the knowledge of (A_{t12}, A_{t34}, E).

We will proceed in the following with the resolution of the linear system of equations (37,38,39) to find the metric components G_{11}, G_{13} and G_{33}. For this, we need to assume an ansätz for each function in the set (A_{t12}, A_{t34}, E). By examining the special supergravity solutions of Section (3.1), Section (3.2) and Section (3.3), we arrive to these ansätze for the background fields (A_{t12}, A_{t34}, E):

$$A_{t12} = \frac{p_{11}\, h_1 + p_{22}\, h_2 + p_{12}\, h_1\, h_2}{E}\,, \tag{43}$$

$$A_{t34} = \frac{q_{11}\, h_1 + q_{22}\, h_2 + q_{12}\, h_1\, h_2}{E}\,, \tag{44}$$

$$E = l_{00} + l_{11}\, h_1 + l_{22}\, h_2 + l_{12}\, h_1\, h_2\,. \tag{45}$$

To determine the coefficients (q_{ab}, p_{ab}, l_{ab}), $a, b = 1, 2$, we refer back again to the special limit of two parallel D2-branes, two orthogonal D2-branes and a single rotated D2-brane given in (7,10,13), respectively. Then by a straightforward comparison with these special, we make the following identifications:

$$\begin{aligned} p_{11} &= \cos^2\theta\,, & p_{22} &= 1\,, \\ q_{11} &= -\sin^2\theta\,, & q_{22} &= 0\,, \\ l_{00} &= l_{11} = l_{22} = 1\,, \\ p_{12} &= -q_{12} = l_{12} \equiv l(\theta)\,. \end{aligned} \tag{46}$$

After replacing by the coefficients above into equations (41,42,45), we obtain

$$\begin{aligned} \partial_1 E &= 1 + l(\theta)\, h_2\,, \quad \partial_2 E = 1 + l(\theta)\, h_1\,, \quad \partial_1 E \partial_2 E - E \partial_1 \partial_2 E = 1 - l(\theta)\,, \\ E &= 1 + h_1 + h_2 + l(\theta)\, h_1\, h_2\,, \\ (\mathcal{A}_1)^2 &= \cos^2\theta + h_2 \left[l(\theta) - \sin^2\theta\right]\,, \end{aligned}$$

$$
\begin{aligned}
(\mathcal{A}_2)^2 &= \left(1 + h_1 \sin^2 \theta\right) \left[1 + h_1\, l\,(\theta)\right] , \\
(\mathcal{B}_1)^2 &= (1 + h_2) \left[\sin^2 \theta + l\,(\theta)\, h_2\right] , \\
(\mathcal{B}_2)^2 &= (h_1 \cos \theta)^2\, l\,(\theta) + h_1 \left[l\,(\theta) - \sin^2 \theta\right] , \\
(\mathcal{C}_1)^2 &= \mathcal{A}_1\, \mathcal{B}_1 , \qquad (\mathcal{C}_2)^2 = \mathcal{A}_2\, \mathcal{B}_2 .
\end{aligned}
\tag{47}
$$

It is clear that we have not yet identified completely the metric components G_{11}, G_{13} and G_{33} in (40) since they still depend on the unknown function $l\,(\theta)$ through the parameters $(\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2, E)$. To find the function $l\,(\theta)$, we use the relations in (47) to substitute by (U, V) of (32), (S_{11}, S_{12}, S_{22}) of (29,30,31) and by (G_{11}, G_{13}, G_{33}) of (40) into the equation of motion of (28). Requiring consistency of this equation of motion (after a considerable amount of algebra) yields the simple answer

$$
l\,(\theta) = \sin^2 \theta .
\tag{48}
$$

Our metric and background fields are thus finally given by

$$
\begin{aligned}
E &= 1 + h_1 + h_2 + \sin^2 \theta\, h_1\, h_2 , \\[2mm]
A &= \frac{h_2 + \cos^2 \theta h_1 + \sin^2 \theta h_1 h_2}{E}\, \mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 - \frac{\sin^2 \theta\,(h_1 + h_1 h_2)}{E}\, \mathrm{d}t \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 \\
&\quad + \frac{\cos \theta \sin \theta h_1}{E} \left(\mathrm{d}t \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 + \mathrm{d}t \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^4\right) , \\
U^2 &= E^{-5/8} , \qquad V^2 = E^{3/8} , \qquad e^{2\phi} = \sqrt{E} , \\
G_{13} &= E^{3/8}\, \frac{\cos \theta\, \sin \theta\, h_1}{E} , \\
G_{11} &= E^{3/8}\, \frac{1 + \sin^2 \theta\, h_1}{E} , \\
G_{33} &= E^{3/8}\, \frac{1 + h_2 + \cos^2 \theta\, h_1}{E} .
\end{aligned}
\tag{49}
$$

This completes our description of the solution of the bound state of two D2-branes at angles which is seen to match the one guessed (but not derived) by the authors of [9]. The advantage of our approach here is that we tried to rely as much as possible only on the equations of motion and the knowledge of some well know limits of the general bound state with angle. Of course the configuration of D2-branes at angles is simple enough that its background fields can be written down without really having to solve any field equations. For more complicated (and higer dimensional) D-brane bound states, however, a pure guess work would not be enough. In this respect, he D2-brane bound state with angle treated here by solving directly the equation serves as a 'theoretical laboratory' to illustrate the kind of calculation that is involved in the general case. Indeed, in the next

section we will take this step further and apply what we have learned from dealing with the D2-brane example to the case of D4-branes at angles. Our conclusion will be that a similar calculation can be applied for any similar Dp-branes at angles.

5   Generalization to D4-Branes at Angles

In this section, we show how the previous calculation can be adapted to account for the case of a bound state of two D4-branes intersecting at non-trivial angles. As for the case of D2-branes, we start by writing down the equations of motion of a D4-brane. They are given by

$$R^\mu{}_\nu = \frac{1}{2}\,\partial^\mu\phi\,\partial_\nu\phi + \frac{1}{2\,4!}\,e^{-\phi/2}\left[4\,F^{\mu\alpha_2\alpha_3\alpha_4}\,F_{\nu\alpha_2\alpha_3\alpha_4} - \frac{3}{8}\,G^\mu{}_\nu\,F^2\right],\tag{50}$$

$$\Box\phi = \frac{1}{4\,4!}\,e^{-\phi/2}\,F^2\,,\tag{51}$$

$$\partial_{\mu_1}\left(\sqrt{-G}\,e^{-\phi/2}\,F^{\mu_1\mu_2\mu_3\mu_4}\right) = 0\,,\tag{52}$$

$$\partial_{[\nu}\,F_{\mu_1\mu_2\mu_3\mu_4]} = 0\,.\tag{53}$$

The last equation is the Bianchi identity $dF = 0$ and we note that the D4-brane is *magnetically* charged.

One can repeat the same steps done before for the D2-brane case and begin first by constructing the configuration of two D4-branes related by $SU(2)$ rotations. We take one of the D4-branes, which is described by the harmonic function h_1, to be parallel to $(y^1, y^2, x^3, x^4)-$directions. The worldvolume of the other D4-brane, which is represented by the harmonic function h_2, is chosen to oriented along the $(x^3, x^4, x^6, x^7)-$axes. For later reference and to also facilitate the comparison of the bound state of D4-branes at angles to well known solutions we define the $y^{1,2,5,6}$ rotated axes as follows:

$$\begin{aligned} y^1 &= \cos\theta\,x^1 - \sin\theta\,x^5\,, & y^5 &= \sin\theta\,x^1 + \cos\theta\,x^5\,,\\ y^2 &= \cos\theta\,x^2 + \sin\theta\,x^6\,, & y^6 &= -\sin\theta\,x^2 + \cos\theta\,x^6\,. \end{aligned}\tag{54}$$

In the next step we explore the bound state of D4-branes with angle in the special limits of two parallel D4-brane, two orthogonal D4-branes intersecting over a $2-$brane, and finally that of a single 'tilted' D4-brane. The knowledge of the background fields in these configurations will allow us to extract the suitable ansätze that we later substitute into the equations of motion (50–53). We do not show here all the details (since they are similar to D2-brane case) but we content ourselves by giving only the final results which (in the Einstein-frame metric) are given by:

$$\mathrm{d}s^2 = U^2\left(-\mathrm{d}t^2 + \left(\mathrm{d}x^3\right)^2 + \left(\mathrm{d}x^4\right)^2\right) + G_{11}\left(\left(\mathrm{d}x^1\right)^2 + \left(\mathrm{d}x^2\right)^2\right)$$

$$+ G_{55} \left(\left(\mathrm{d}x^5 \right)^2 + \left(\mathrm{d}x^6 \right)^2 \right) + 2\, G_{15} \left(\mathrm{d}x^1\, \mathrm{d}x^5 - \mathrm{d}x^2\, \mathrm{d}x^6 \right) + V^2 \sum_{i=7,8,9} \left(\mathrm{d}x^i \right)^2 ,$$

$$
\begin{aligned}
{}^\star F \;=\; & -\, \partial_i \left[\frac{h_2 + \cos^2 \theta h_1 + l\left(\theta\right) h_1 h_2}{E} \right] \mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 \wedge \mathrm{d}x^i \\
& +\, \partial_i \left[\frac{\sin^2 \theta h_1 + l\left(\theta\right) h_1 h_2}{E} \right] \mathrm{d}t \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6 \wedge \mathrm{d}x^i \\
& -\, \partial_i \left[\frac{\cos \theta \sin \theta h_1}{E} \right] \mathrm{d}t \wedge \left(\mathrm{d}x^2 \wedge \mathrm{d}x^5 + \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^6 \right) \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 \wedge \mathrm{d}x^i ,
\end{aligned}
$$

$$e^{-2\phi} \;=\; \sqrt{1 + h_1 + h_2 + l\left(\theta\right) h_1 h_2} \; . \tag{55}$$

As in the D2-brane case, we have

$$
\begin{aligned}
U \;&\equiv\; U\left(h_1, h_2, \theta\right), \;\; V \equiv V\left(h_1, h_2, \theta\right), \;\; E \equiv E\left(h_1, h_2, \theta\right), \\
G_{11} \;&\equiv\; G_{11}\left(h_1, h_2, \theta\right), \;\; G_{13} \equiv G_{13}\left(h_1, h_2, \theta\right), \;\; G_{33} \equiv G_{33}\left(h_1, h_2, \theta\right), \\
\mathcal{A}_m \;&\equiv\; \mathcal{A}_m\left(h_1, h_2, \theta\right), \;\; \mathcal{B}_m \equiv \mathcal{B}_m\left(h_1, h_2, \theta\right), \;\; \mathcal{C}_m \equiv \mathcal{C}_m\left(h_1, h_2, \theta\right), \;\; m = 1, 2 , \tag{56}
\end{aligned}
$$

and the harmonic functions become $h_{1,2} = c_{1,2}\, Q_{1,2} / |\vec{r} - \vec{r}_{1,2}|$, with $r^2 = \sum_{i=7}^{9} (x^i)^2$.

In order to determine the set of functions $(U, V, G_{11}, G_{13}, G_{33}, l\left(\theta\right))$, we replace by the above ansätze into the field equations of motion (50–53). The equations of motion are found to take the simple form:

$$\partial_i \partial_i \phi \;=\; \frac{1}{4} \frac{1}{4!}\, V^2\, e^{-\phi/2}\, F^2 = \frac{4}{3}\, \partial_i \partial_i \ln U = -\, \partial_i \partial_i \ln \left(G_{11} G_{55} - G_{15}^2 \right) , \tag{57}$$

$$
\begin{aligned}
& \partial_i \ln U\, \partial_j \ln U - \frac{1}{2} \left(\partial_i G^{11}\, \partial_j G_{11} + \partial_i G^{55}\, \partial_j G_{55} + 2 \partial_i G^{15}\, \partial_j G_{15} \right) + \partial_i \ln V\, \partial_j \ln V \\
& + \delta_{ij}\, \partial_k \partial_k \ln V = -\frac{1}{2} \partial_i \phi\, \partial_j \phi + \frac{3}{8}\, V^2\, \delta_{ij}\, \frac{1}{4!}\, e^{-\phi/2}\, F^2 - 2\, \frac{1}{4!}\, e^{-\phi/2}\, \left(F^2 \right)_{ij} . \tag{58}
\end{aligned}
$$

Equation (57) leads readily to the solutions:

$$U = E^{-3/16} , \qquad G_{11}\, G_{55} - G_{15}^2 = E^{1/4} . \tag{59}$$

Next to determine V we need to use the extremality condition which follows from the requirement that the bound state of angled D4-branes preserves $1/4$ of supersymmetries. An analysis similar to the D2-brane example leads in the present case to this constraint

$$U^3 \left(G_{11}\, G_{55} - G_{15}^2 \right) V = 1 , \tag{60}$$

which readily yields

$$V = E^{5/16} . \tag{61}$$

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To continue with the identification of the rest of the background fields of the angled D4-branes, we need to calculate the quantity F^2 from the ansätz field strength of (55). We skip here all the details involved in the evaluation of F^2 but it is clear that at the end it will turn out to be a function of $(h_1, h_2, l(\theta))$. Substituting then by its expression into the two equations of motion (57,58), (and without giving any details since they are identical to those explained in Section (4)), we are lead to the rest of the supergravity solution which is found to be:

$$
\begin{aligned}
l(\theta) &= \sin^2\theta \,, \qquad E = 1 + h_1 + h_2 + \sin^2\theta\, h_1 h_2 \,, \\
e^{-2\phi} &= \sqrt{E} \,, \quad U^2 = E^{-3/8} \,, \quad V^2 = E^{5/8} \,,
\end{aligned}
$$

$$
\begin{aligned}
G_{15} &= E^{-3/8}\, \frac{\cos\theta\,\sin\theta\, h_1}{E} \,, \\
G_{11} &= E^{-3/8}\, \frac{1 + \sin^2\theta\, h_1}{E} \,, \\
G_{55} &= E^{-3/8}\, \frac{1 + h_2 + \cos^2\theta\, h_1}{E} \,, \\
\star F &= -\partial_i \left[\frac{h_2 + \cos^2\theta h_1 + \sin^2\theta h_1 h_2}{E} \right] \mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 \wedge \mathrm{d}x^i \\
&\quad + \partial_i \left[\frac{\sin^2\theta\,(h_1 + h_1 h_2)}{E} \right] \mathrm{d}t \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6 \wedge \mathrm{d}x^i \\
&\quad - \partial_i \left[\frac{\cos\theta\,\sin\theta h_1}{E} \right] \mathrm{d}t \wedge \left(\mathrm{d}x^2 \wedge \mathrm{d}x^5 + \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^6 \right) \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 \wedge \mathrm{d}x^i \,. \quad (62)
\end{aligned}
$$

So with the above background fields our 1/4 supersymmetric bound state of angled two D4-branes is totally defined. The supergravity solution of this configuration was also conidered in [8,9].

6 Conclusions

In this paper, we considered the supergravity solutions describing D2-brane and D4-brane bound states where the constituent D-branes are oriented at non-trivial angles with respect to one another. Since the studied bound states are taken to saturate a BPS bound where 1/4 of supersymmetries are preserved, the constituent D-branes must be related by $SU(2)$ rotations. The originality of our work here resides in our use of the equations of motion to solve directly for the background fields of the angled D-branes. In fact, using always only the equations of motion, the calculation of this paper can be applied easily to derive the low-energy solution of any configuration of multiple Dp-branes oriented at angles. Finally, the work in this paper will serve as a basis to derive the supregravity classical solutions of more general bound states with angles in type II string theory and M-theory [18].

Acknowledgments

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